## Cambridge IGCSE ${ }^{\text {TM }}$

CANDIDATE NAME

CENTRE


## ADDITIONAL MATHEMATICS

Paper 1
May/June 2021

You must answer on the question paper.
No additional materials are needed.

## INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.


## INFORMATION

- The total mark for this paper is 80 .
- The number of marks for each question or part question is shown in brackets [ ].


## Mathematical Formulae

## 1. ALGEBRA

## Quadratic Equation

For the equation $a x^{2}+b x+c=0$,

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

## Binomial Theorem

$$
(a+b)^{n}=a^{n}+\binom{n}{1} a^{n-1} b+\binom{n}{2} a^{n-2} b^{2}+\ldots+\binom{n}{r} a^{n-r} b^{r}+\ldots+b^{n}
$$

where $n$ is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!r!}$

Arithmetic series

$$
\begin{aligned}
& u_{n}=a+(n-1) d \\
& S_{n}=\frac{1}{2} n(a+l)=\frac{1}{2} n\{2 a+(n-1) d\}
\end{aligned}
$$

Geometric series

$$
\begin{aligned}
& u_{n}=a r^{n-1} \\
& S_{n}=\frac{a\left(1-r^{n}\right)}{1-r}(r \neq 1) \\
& S_{\infty}=\frac{a}{1-r}(|r|<1)
\end{aligned}
$$

## 2. TRIGONOMETRY

## Identities

$$
\begin{gathered}
\sin ^{2} A+\cos ^{2} A=1 \\
\sec ^{2} A=1+\tan ^{2} A \\
\operatorname{cosec}^{2} A=1+\cot ^{2} A
\end{gathered}
$$

Formulae for $\triangle A B C$

$$
\begin{gathered}
\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
a^{2}=b^{2}+c^{2}-2 b c \cos A \\
\Delta=\frac{1}{2} b c \sin A
\end{gathered}
$$

1 (a) On the axes, sketch the graph of $y=5(x+1)(3 x-2)(x-2)$, stating the intercepts with the coordinate axes.

(b) Hence find the values of $x$ for which $5(x+1)(3 x-2)(x-2)>0$.

2 Find $\int_{3}^{5}\left(\frac{1}{x-1}-\frac{1}{(x-1)^{2}}\right) \mathrm{d} x$, giving your answer in the form $a+\ln b$, where $a$ and $b$ are rational numbers.

3 The polynomial $\mathrm{p}(x)=a x^{3}-9 x^{2}+b x-6$, where $a$ and $b$ are constants, has a factor of $x-2$. The polynomial has a remainder of 66 when divided by $x-3$.
(a) Find the value of $a$ and of $b$.
(b) Using your values of $a$ and $b$, show that $\mathrm{p}(x)=(x-2) \mathrm{q}(x)$, where $\mathrm{q}(x)$ is a quadratic factor to be found.
(c) Hence show that the equation $\mathrm{p}(x)=0$ has only one real solution.

4 The first 3 terms in the expansion of $(a+x)^{3}\left(1-\frac{x}{3}\right)^{5}$, in ascending powers of $x$, can be written in the form $27+b x+c x^{2}$, where $a, b$ and $c$ are integers. Find the values of $a, b$ and $c$.

5 The functions $f$ and $g$ are defined as follows.

$$
\begin{array}{ll}
\mathrm{f}(x)=x^{2}+4 x & \text { for } x \in \mathbb{R} \\
\mathrm{~g}(x)=1+\mathrm{e}^{2 x} & \text { for } x \in \mathbb{R}
\end{array}
$$

(a) Find the range of f .
(b) Write down the range of g .
(c) Find the exact solution of the equation $\operatorname{fg}(x)=21$, giving your answer as a single logarithm. [4]

6 (a) (i) Find how many different 5-digit numbers can be formed using the digits 1, 3, 5, 6, 8 and 9 . No digit may be used more than once in any 5 -digit number.
(ii) How many of these 5 -digit numbers are odd?
(iii) How many of these 5-digit numbers are odd and greater than 60000 ?
(b) Given that $45 \times{ }^{n} \mathrm{C}_{4}=(n+1) \times{ }^{n+1} \mathrm{C}_{5}$, find the value of $n$.

7 (a) In this question, all lengths are in metres and time, $t$, is in seconds.


The diagram shows the displacement-time graph for a runner, for $0 \leqslant t \leqslant 40$.
(i) Find the distance the runner has travelled when $t=40$.
(ii) On the axes, draw the corresponding velocity-time graph for the runner, for $0 \leqslant t \leqslant 40$.

(b) A particle, $P$, moves in a straight line such that its displacement from a fixed point at time $t$ is $s$. The acceleration of $P$ is given by $(2 t+4)^{-\frac{1}{2}}$, for $t>0$.
(i) Given that $P$ has a velocity of 9 when $t=6$, find the velocity of $P$ at time $t$.
(ii) Given that $s=\frac{1}{3}$ when $t=6$, find the displacement of $P$ at time $t$.

## 8 DO NOT USE A CALCULATOR IN THIS QUESTION.

A curve has equation $y=(2-\sqrt{3}) x^{2}+x-1$. The $x$-coordinate of a point $A$ on the curve is $\frac{\sqrt{3}+1}{2-\sqrt{3}}$.
(a) Show that the coordinates of $A$ can be written in the form $(p+q \sqrt{3}, r+s \sqrt{3})$, where $p, q, r$ and $s$ are integers.
(b) Find the $x$-coordinate of the stationary point on the curve, giving your answer in the form $a+b \sqrt{3}$, where $a$ and $b$ are rational numbers.

9 (a) (i) Write $6 x y+3 y+4 x+2$ in the form $(a x+b)(c y+d)$, where $a, b, c$ and $d$ are positive integers.
(ii) Hence solve the equation $6 \sin \theta \cos \theta+3 \cos \theta+4 \sin \theta+2=0$ for $0^{\circ}<\theta<360^{\circ}$.
(b) Solve the equation $\frac{1}{2} \sec \left(2 \phi+\frac{\pi}{4}\right)=\frac{1}{\sqrt{3}}$ for $-\pi<\phi<\pi$, where $\phi$ is in radians. Give your answers in terms of $\pi$.

10 In this question all lengths are in centimetres.


The diagram shows a shaded shape. The arc $A B$ is the major arc of a circle, centre $O$, radius 10 . The line $A B$ is of length 15 , the line $O C$ is of length 25 and the lengths of $A C$ and $B C$ are equal.
(a) Show that the angle $A O B$ is 1.70 radians correct to 2 decimal places.
(b) Find the perimeter of the shaded shape.
(c) Find the area of the shaded shape.

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